DOCUMENT RESUME

ED 408 160		SE 060 194

AUTHOR	Bolte, Linda A.
TITLE	Assessing Mathematical Knowledge with Concept Maps and Interpretive Essays.
PUB DATE	24 Mar 97
NOTE	28p.; Paper presented at the Annual Meeting of the American Educational Research Association (Chicago, IL, March 24, 1997).
PUB TYPE	Reports - Research (143) -- Speeches/Meeting Papers (150)
EDRS PRICE	MF01/PC02 Plus Postage.
DESCRIPTORS	Concept Formation; *Concept Mapping; Elementary Secondary Education; Knowledge Level; *Knowledge Representation; Learning Strategies; *Mathematical Concepts; Mathematics Curriculum; Mathematics Instruction; Preservice Teacher Education; Problem Solving; Student Evaluation; *Writing Across the Curriculum

ABSTRACT

This study explores the combined use of concept maps and interpretive essays as a method of assessment in three mathematics courses. The primary objectives were to describe and document: (1) the use of concept maps and written essays to assess the connectedness of students' knowledge; (2) the correlation between students' scores on the concept maps and written essays, course exams, and final grades; and (3) the degree to which learning was enhanced through the use of concept maps and written essays. The subjects included prospective elementary teachers (N=23), calculus students (N=63), and prospective secondary mathematics teachers (N=17). Results indicate that concept maps combined with written essays are viable tools for enhancing and assessing students' organization of mathematical knowledge. Contains 24 references. (DDR)

Assessing Mathematical Knowledge With Concept Maps And Interpretive Essays

(Paper to accompany Poster Session presented at AERA Annual Meeting, March 24, 1997)

Linda A. Bolte
Eastern Washington University
Cheney, Washington

Running head:
Assessing Mathematical Knowledge

Introduction

The development of an integrated knowledge-base in mathematics and the communication of mathematical knowledge are two aspects of a meaningful mathematical experience that are strongly emphasized in the current literature (NCTM, 1989; Roberts, 1996; Steen, 1989). These documents assert that exploring mathematical connections and viewing mathematics as communication are essential components of a strong mathematics curriculum. This entails allowing students to investigate the connections and interplay among various mathematical topics so they can recognize equivalent representations of the same concept; and providing students with the opportunity to reflect on and clarify their own thinking about mathematical ideas and situations, and express mathematical ideas in writing.

Although widely accepted as worthwhile, this notion is largely based on intuition (Skemp, 1978). Direct measurement of students' organization of mathematical knowledge, the effect of this organization on their learning, and the use of concept maps as an assessment instrument in mathematics are relatively unexplored areas of research. This study explored the combined use of concept maps and interpretive essays as a method of assessment in three mathematics courses. The primary objectives of this study were to describe and document: 1) the use of concept maps and written essays to assess the connectedness of students' knowledge; 2) the correlation between students' scores on the concept maps and written essays, course exams, and final grade; and 3) the degree to which learning was enhanced by the use of concept maps and written essays. Results indicated that concept maps, when combined

with written essays, are viable tools for enhancing and assessing students' organization of mathematical knowledge.

Conceptual Base

The use of concept maps as an indication of the connectedness of knowledge is largely based on the work of Novak (1984) in the area of science. As graphical representations that involve linking related concepts to form chains of relationships, concept maps have been "developed specifically to tap into a learner's cognitive structure and to externalize, for both the learner and the teacher to see, what the learner already knows" (Novak, 1984, p. 40). In a concept map, related concepts are represented as nodes and the specific relationship between two concepts is indicated by linking words that are written along the line connecting the nodes (see Appendix A for sample maps).

The use of concept maps to assess the organization of mathematical knowledge is closely tied to a model developed by Hiebert and Carpenter (1992) for analyzing the issues of learning and teaching mathematics with understanding. Within this model, which is based on the assumption that internal representations of knowledge are connected in some useful way, networks of knowledge are generally described metaphorically in two ways: as vertical hierarchies in which some representations subsume other representations and as web-like arrangements in which pieces of information and corresponding relationships between the items of information form simple linear chains or complex networks.

Educational applications of concept mapping include their use as a learning/study strategy and as an assessment instrument in a variety of settings (Bartels, 1995; Beyerbach, 1988; Mansfield & Happs, 1991; Novak, 1991). As noted by Angelo and Cross, the use of concept maps "stimulate students to create, and allow faculty to assess, original intellectual products that result from a synthesis of the course content and the students' intelligence, judgment, knowledge, and skills" (1993, p. 181). Concept maps have also been used effectively within an instructional setting to identify student misconceptions (Bartels, 1995; Kounba, 1994; Novak, 1984).

The use of writing as a means of enhancing and assessing student understanding is consistent with the current trend to incorporate writing in mathematics classes (Connolly & Vilardi, 1989; Sterrett, 1990). Although there are a substantial number of proponents of writing, research which verifies the benefits of writing as a means to enhance and assess mathematical understanding is somewhat limited and focuses primarily on the use of journals or writing prompts. (Borasi & Rose, 1989; Clarke, 1993; Miller, 1992; Miller & England, 1989; Powell & Lopez, 1989; Swinson, 1992). Of particular interest is the use of samples of students' writing to identify misconceptions (Birken, 1989; Miller, 1992).

Design of Study

The subjects who participated in this study consisted of the following groups of students at a regional state university in the Northwest: 23 prospective elementary teachers enrolled in the first of two required mathematics content courses, Structure of

Mathematics I; 63 students enrolled in three sections of Calculus I; and 17 prospective secondary mathematics teachers enrolled in a required course, Survey of Geometries.

Students were introduced to the construction of concept maps and writing the accompanying interpretive essay during a regular class meeting. Using a list of words related to a familiar topic, they were led through the multi-step process of constructing a concept map as a class, with each step illustrated on an overhead transparency. First, students were instructed to read the list of terms in order to become familiar with the general topic being explored. Next, the terms were sorted into clusters according to the extent to which they are related. At this point, it was stressed that sorting the terms is a very individualized process; each student should construct a map that makes sense to him/her as a means of constructing knowledge. There was no one, correct map to be constructed; students could omit any terms they were unable to use in their map or add additional terms. Once the terms were sorted, they were arranged in clusters around a central concept. Because an arrangement may be either hierarchical or web-like, depending on how one views the relationships between the terms, students were shown examples of both types of maps. Students were encouraged to explore several arrangements of the terms until satisfied with the organization of their map. Next, linking lines between terms in individual clusters and cross-links between related clusters were drawn. The final step consisted of labeling the linking lines to explain the relationships being illustrated. Because the inclusion of linking words is critical to interpretation of the concept map and students find it difficult to specify some relationships they wish to express on the map, this aspect of construction was discussed at some length. The

class generated possible linking words (e.g., has the property, is an example of, involves) and discussed the need to draw directional arrows on the linking lines to indicate the direction of the relationship being expressed.

Following this introduction, students were given a list of 20–30 words related to the selected topic and asked to construct a concept map based on these terms. They were encouraged to use outside resources, as needed, to clarify the meaning of unknown terms (see Appendix B for the list of terms used in each class).

Once their concept map was completed, students wrote an accompanying interpretive essay, in which they clarified and developed the relationships expressed on the map. These essays were not limited to merely explaining how the map was organized; they were meant to give students the opportunity to reflect on the relationships illustrated on their concept map and refine their thoughts. Because many students were not comfortable writing about mathematics, a few broad guidelines were provided: make your paper reader-friendly, perhaps thinking of a classmate as the reader rather than the instructor; be as thorough in your discussion as possible, including additional information you think is relevant and your personal insights; and consider personalizing your essay, for example, make it into a story or set it in a creative context.

Some students were uncomfortable with the nature of the task, and expressed a concern as to whether they were "doing it right" or if there was a prescribed length for the essay. These students were encouraged to focus on creating a map that meant something to them. No page limit was imposed on the essay; the only requirement was that it effectively communicate the connections they perceived to the reader.

Merely listing textbook definitions or a series of disjointed statements was strongly discouraged (see Appendix C for excerpts from sample essays).

Each student in Structure of Mathematics I completed two concept maps, one on sets and functions and the other on number theory, and wrote accompanying interpretive essays after completing the chapter dealing with each topic. Each student in Calculus I completed two concept maps and interpretive essays, one on functions as a review of important concepts at the beginning of the course and the other on differential and integral calculus as a summative activity at the end of the course. Students in Survey of Geometries completed one concept map based on terms related to finite, Euclidean and non-Euclidean geometries and wrote an accompanying interpretive essay at the end of the course.

Each concept map and interpretive essay was scored using a holistic scoring criteria. The concept map criteria focus on organization and accuracy, whereas the interpretive essay criteria focus on communication, organization, and mechanics. Students were given the scoring criteria prior to beginning the activity to allow them the opportunity to self-evaluate their work. This holistic scoring criteria, similar to the one proposed by Bartels (1995), streamlines the more cumbersome, formal grading schemes that are sometimes associated with concept maps (Angelo & Cross, 1993) (see Appendix D for the scoring criteria).

At the end of each course, students were asked to comment on the use of concept maps and interpretive essays, specifically addressing the advantages and disadvantages of this type of activity. The final component of the study entailed comparing individual student performance on homework and quizzes, hourly tests, the final

exam, and course grade with the scores on the concept maps and written interpretive essays.

Results

Assessment of Connectedness of Mathematical Knowledge

The concept maps constructed in each course explicitly depicted the mathematical connections each student perceived. Students' basic understanding of the terms, as well as their ability to reflect on the deeper meaning of the terms and the flexible use of the mathematical terms were assessed. While many students initially had a basic understanding of what each term meant, as they attempted to show and write about the interrelationships between the terms, they were required to look at the meaning of the terms from different perspectives. For example, on the sets and functions map from Structure of Mathematics I, some students were able to illustrate the relationships between the terms associated with sets and those associated with relations and functions, but were unable to define a connection between these clusters. On the functions map in Calculus I, the terms exponential and logarithmic refer to types of functions, but should also be connected to inverse; on the second map, some students were able to illustrate a relationship between calculus and limits, but were unable to define a connection between limit and derivative and integral. The extent to which a student was able to make these connections allowed for a more precise evaluation of what the student knew, of what the student did not know, and of misconceptions that existed.

Many misconceptions were readily identified by noting inappropriate or missing linking words, the omission of terms or linking lines, and overall organization of the concept map. For example, several Calculus students indicated circles and ellipses are functions, while others indicated both the definite and indefinite integral find the area under a curve. On the Survey of Geometries maps, some students did not illustrate any connection between Birkhoff's, Hilbert's, and the SMSG's axiomatic systems for Euclidean geometry, but showed them as completely unrelated. Analysis of the interpretive essay corroborated these misconceptions or indicated the error was merely an inaccuracy or omission in the construction of the concept map. A common error shown on the functions concept map was illustrating that all functions are one-to-one. However, frequently in the context of the essay, the student distinguished between the defining characteristic of a function, single-valuedness, and the property one-to one. Likewise, several students stated in the essay that differentiation is the inverse of integration, but did not illustrate this relationship on their map.

As a means of assessment, this dual approach had benefits beyond using each method individually; by relying on both visual and verbal communication of mathematical knowledge, it resulted in a more extensive representation of students' mathematical knowledge. Creating the map gave an underlying structure to the essay without being overly rigid; because students were allowed the freedom to set the essay in any context, it tended to more personal and informal than purely technical writing. By the same token, the essay justified the organization of the concept map and clarified possible technical inaccuracies in the construction of the map, such as inappropriate or missing linking lines. It allowed the students to expand on the connections they saw

between the terms, to explain the connections they perceived in a specific context, and to refine their thinking.

Correlation With Traditional Means of Assessment

The combined concept map/interpretive essay scores were highly correlated with the other more traditional measures used in the courses as shown below. Based on this data, the combined concept map/essay scores were more highly correlated with the final grade than the combined homework and quiz scores. Although hourly tests and the final exam score were more highly correlated with the final grade than the concept map/essay scores, these measures were more highly weighted in the final grade than the concept map/essay score. The correlations were lowest for Structure of Mathematics I; however, in this course the topics used for the two concept maps were not as integral to the overall course content as the topics used for Calculus I and Survey of Geometries.

	Calculus I–Fall '96	Calculus I–Fall '97	Survey of Geometries	Structure of Math
Map/Essay and Grade	0.86	0.88	0.74	0.65
Homework/Quizzes and Grade	0.75	0.76	0.72	0.66
Tests and Grade	0.96	0.98	0.95	0.96
Final Exam and Grade	0.93	0.87	0.96	0.93
Maps/Essays and Tests/Final	0.83	0.82	0.72	0.61

Enhancement of Student Learning

There were numerous ways in which the use of concept maps/interpretive essays enhanced the learning of mathematics: 1) students were able to reflect on their work because the mathematical connections were explicitly depicted; 2) students could modify and extend their knowledge in the process of constructing the map; and 3) students experienced mathematics as a creative activity.

Due to its open-ended tone, this activity helped dispel some dysfunctional mathematical beliefs. For example, in many students' conception of mathematics, the scope of mathematical activity is limited to problems that are well defined and have an exact and predetermined solution; mathematical activity is characterized as the recalling and applying of learned procedures; and by nature, mathematical knowledge is right or wrong, with no room for personal judgment (Borasi, 1990). These beliefs, which characterize a dualist product-view of mathematics, were challenged by engaging students in a learning situation that requires reflection on past and current knowledge, by encouraging the creative and flexible expression of their ideas, and by not imposing a time restraint on the formulation and communication of their ideas. Not restricting students to a hierarchical structure on the organization of the maps, as Novak proposes (1984), allowed more flexibility and opportunity for creativity than is sometimes the case when using concept maps.

Although they found the construction of the maps time consuming, students generally reported, as a verbal comment or in their essay, that they enjoyed constructing the concept maps and found both activities to be worthwhile. One student wrote: "Concept maps, what a concept! This was a very fun thought provoking

activity. At times I thought that I would never be able to make it work, but perseverance prevailed!" Many students felt the construction of the concept map and writing the interpretive essay encouraged them to reflect on their knowledge and enhanced their integration of this knowledge. Others indicated they enjoyed the opportunity to demonstrate their knowledge in a non-numerical way that allowed them to be creative.

One calculus author began his essay in this way:

> Dirk the Derivative was born into the complex and ruthless Land of Calculus in the late 1600's.... It was a troublesome time in Europe before Dirk was handed the sword and crown he was to rule by. The Curse of Tangent and the Dilemma of Area had plagued the lives of mathematicians for centuries. Dirk, as an instantaneous rate of change, revolutionized math forever. He is regarded as a limit of average velocities over shorter and shorter time intervals. With this power he can solve the most vengeful obstacles math can deliver.
>
> Dirk the Derivative is a function that is derived from other functions. Giving that the original function exists, he can be calculated as a slope of the tangent line at the existing points of a continuous function.... There are evil points within some intervals in which Dirk cannot exist. These points include the Cryptic Corner, Venus of the Vertical Tangent, the Dragon of Discontinuity, and the Knights of Asymptote. When a function is spread out over the Charted Map of Camelot, the land below its curve is knows as the Integral of Enchantment. The area of this land can be calculated using the Riemann Staff, or by employing the wizard of the Definite Integral Tower....

A geometry student who had arranged the terms on nested rectangles of different colors with each color representing specific properties began her essay in this way:

> How does one make the links that describe the different kinds of geometries and their relationship to one another? Follow me and I will lead you through the tightly woven organizations of geometries as I see them. First, notice that many colors make up the model, at first glance this entices the on-looker. After seeing the model they want to know more, they ask themselves, "Could it be? Could math be that beautiful?" Surprised and curious the spectator continues to stare into the model. Now is a great chance, you have an audience, so take this opportunity to sneak in some knowledge.

Although the author of a very creative functions concept map did not write a very thorough essay, he did express strong feelings on the construction of the concept map: "There are also some illustrations which basically serve to define me, and how I do things. I get so sick of being just another one of the same old things, so I make my projects so they can only be put into a category by themselves. Anyways, that's it. Hope you enjoy the ride."

Conclusions

These findings suggest that concept maps, used in conjunction with interpretive essays, are a viable addition to traditional assessment in mathematics classes. The combined use of these instruments provides substantial insight into the degree of connectedness of students' knowledge with respect to the given topics and enables the instructor to assess the degree to which the mathematical material is being integrated

into the learners' knowledge base. By relying on two avenues of expression, students are able to communicate their knowledge in a more complete manner.

The results of this study have significance in the area of mathematics education in a number of ways. They augment the research supporting the recommendations made in the current literature (NCTM, 1989; Roberts, 1996; Steen, 1989), and support the viability of integrating the learning and teaching of mathematics within a classroom setting – a research area of current interest in mathematics education (Research Advisory Committee, 1990; Sierpinska, Kilpatrick, Balacheff, Howson, Sfard, & Steinbring, 1993).

References

Angelo, T., & Cross, K. P. (1993). *Classroom Assessment Techniques: A Handbook for College Teachers*. San Francisco: Jossey-Bass.

Bartels, B. (1995). Promoting mathematics connections with concept mapping. *Mathematics Teaching in the Middle Grades, 1,* 542-549.

Beyerbach, B. (1988). Developing a technical vocabulary on teacher planning: Preservice teachers' concept maps. *Teaching & Teacher Education, 4,* 339-347.

Birken, M. (1989). Using writing to assist learning in college mathematics classes. In P. Connolly & T. Vilardi (Eds.) *Writing to Learn Mathematics and Science* (pp. 33–47). New York: Teachers College Press.

Borasi, R. (1990). The invisible hand operating in mathematics instruction: Students' conceptions and expectations. In N. Webb (Ed.), *Assessment in the Mathematics Classroom . 1993 Yearbook* (pp. 174-181). Reston, VA: NCTM.

Borasi, R., & Rose, B. J. (1989). Journal writing and mathematics instruction. *Educational Studies in Mathematics, 20,* 347-65.

Clarke, D. (1993). Probing the structure of mathematical writing. *Educational Studies in Mathematics, 25, 235-50.*

Connolly, P, & Vilardi, T. (Eds.) (1989). *Writing to Learn Mathematics and Science.* New York: Teachers College Press.

Hiebert, J., & Carpenter, T. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook for research on mathematics teaching and learning* (pp. 65–97). New York: Macmillan.

Kounba, V. (1994). Self-evaluation as an Act of Teaching. *Mathematics Teacher, 87,* 354-58.

Mansfield, H., & Happs, J. (1991). Concept maps. *Australian Mathematics Teacher, 47*(3), 30-33.

Miller, L. D. (1992). Teacher benefits from using impromptu writing prompts in algebra class. *Journal for Research in Mathematics Education, 23,* 329-340

Miller, L. D. & England, D. A. (1989). Writing to learn algebra. *School Science and Mathematics, 89,* 299-312.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: Author.

Novak, J. (1984). *Learning how to learn.* New York: Cambridge University Press.

Novak, J. (1991). Clarify with concept maps. *Science Teacher, 58*(7), 44-49.

Powell, A, & Lopez, J. (1989). Writing as a vehicle to learn mathematics: A case study. In P. Connolly & T. Vilardi (Eds.) *Writing to Learn Mathematics and Science* (pp. 157–177). New York: Teachers College Press.

Research Advisory Committee (1990). Mathematics education reform and mathematics education research: Opportunities, obstacles, and obligations. *Journal for Research in Mathematics Education, 21,* 287-292.

Roberts, W. (Ed.) (1996). *Calculus: The Dynamics of Change.* Washington, D.C.: Mathematical Association of America.

Sierpinska, Kilpatrick, Balacheff, Howson, Sfard, & Steinbring (1993). What is Research in mathematics education, and what are its results? *Journal for Research in Mathematics Education, 24,* 274-278.

Skemp, R. (1978). Relational understanding and instrumental understanding. *Arithmetic Teacher, 25*(3), pp. 9-15.

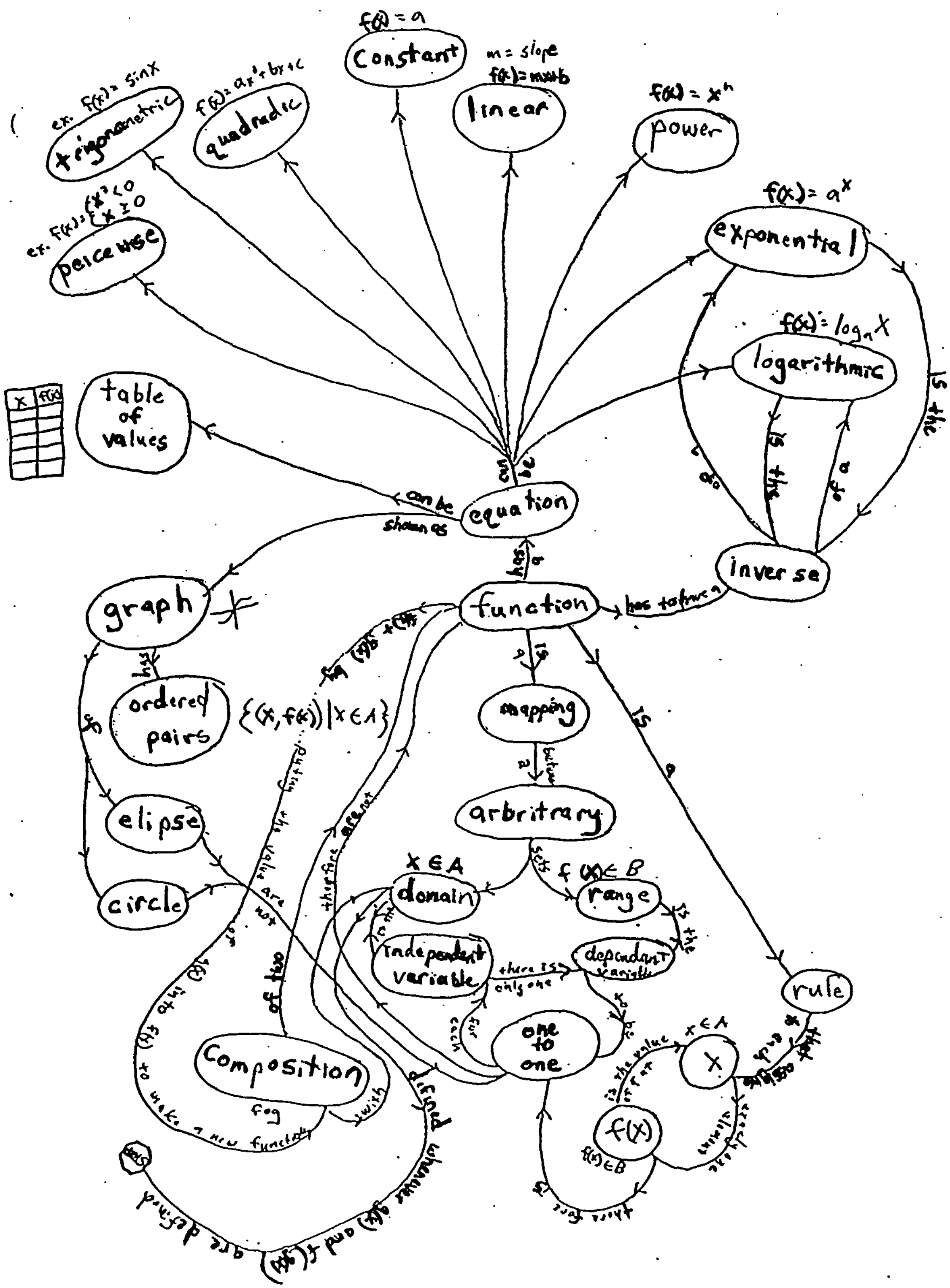
Steen, L. (Ed.) (1989). *Reshaping College Mathematics: A Project of the Committee on the Undergraduate Program in Mathematics.* Washington, D.C.: Mathematical Association of America.

Sterrett (Ed.) (1990). *Using Writing to Teach Mathematics.* Washington, D.C.: Mathematical Association of America.

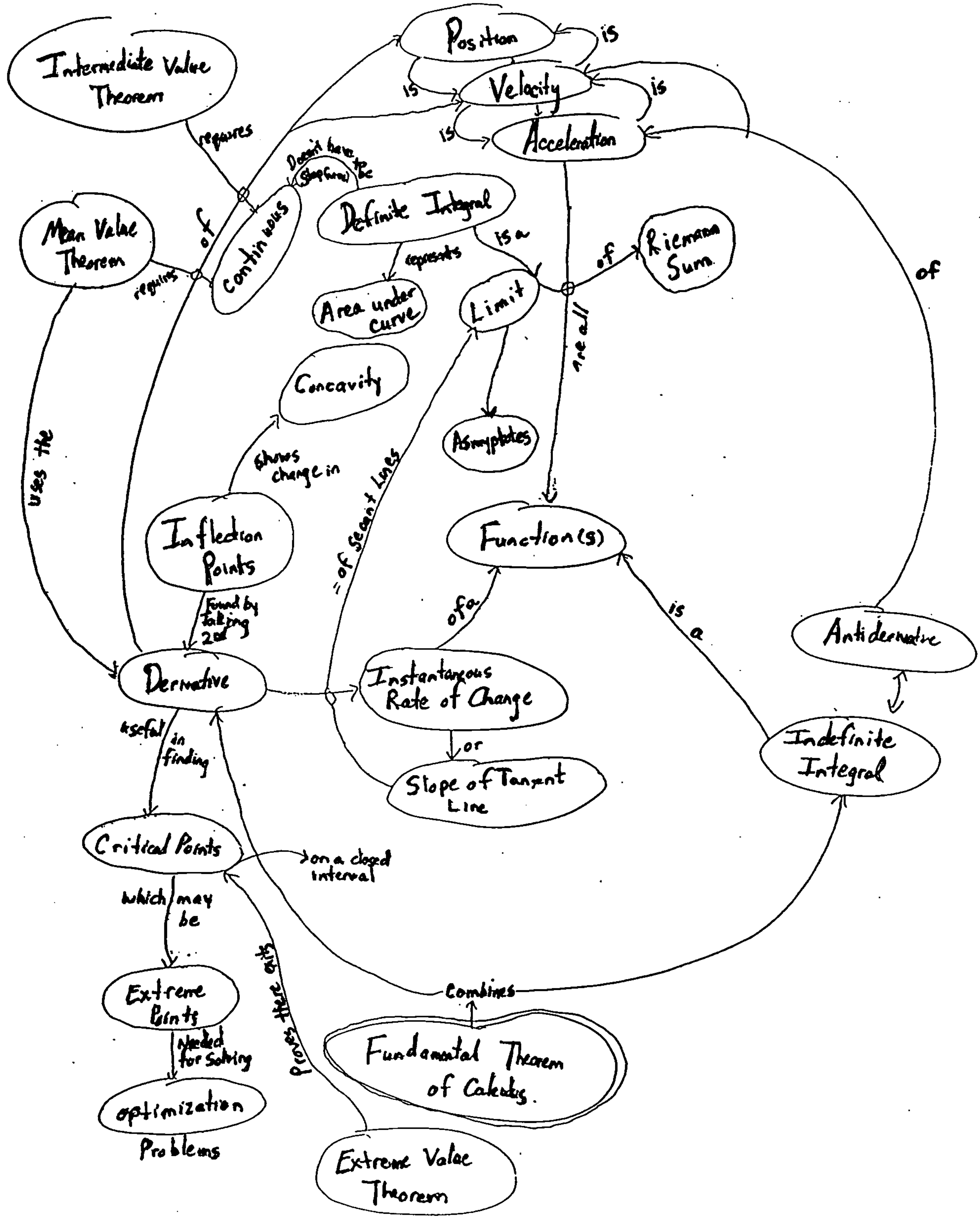
Swinson, K. (1992). Writing activities as strategies for knowledge construction and identification of misconceptions in mathematics. *Journal of Science and Mathematics Education in Southeast Asia, 15*(2), 7-14.

Appendix A: Sample Concept Maps

#1 <u>Calculus I: Functions</u>

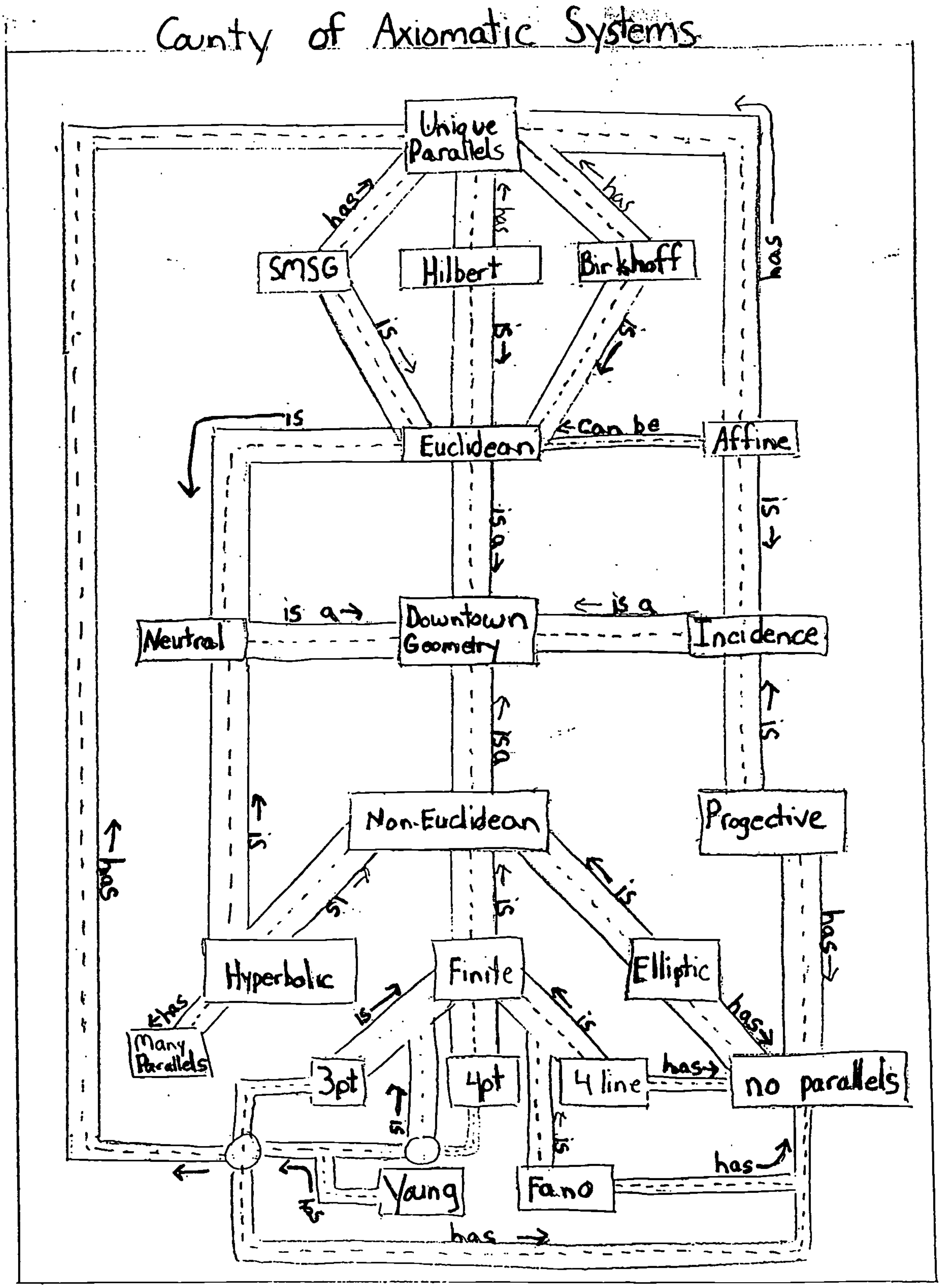


#2 Calculus I: Differential and Integral Calculus



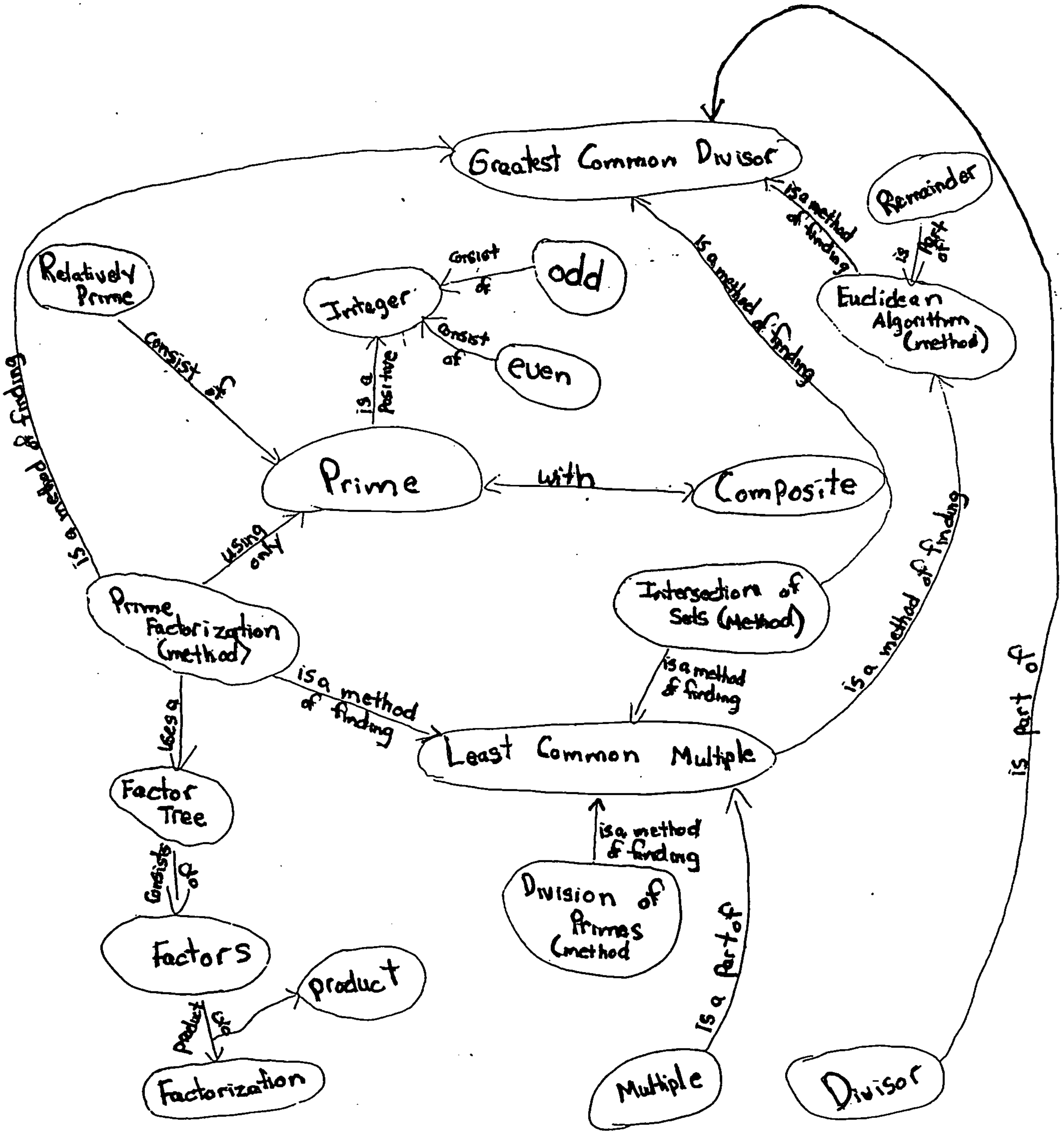
BEST COPY AVAILABLE

#3 Survey of Geometries



BEST COPY AVAILABLE

#4 Structure of Mathematics I: Number Theory



BEST COPY AVAILABLE

Appendix B: Terms Used For Concept Maps

Functions

single-valued	domain	graph	inverse
equation	linear	exponential	quadratic
dependent variable	constant	ellipse	function
trigonometric	composition	arbitrary	range
logarithmic	one-to-one	mapping	rule
independent variable	image	table of values	preimage
ordered pair	piecewise	circle	onto

Differential And Integral Calculus

instantaneous rate of change	continuous	derivative
mean value theorem for integrals	optimization	limit
difference quotient	definite integral	Riemann sum
slope of tangent line	area under curve	asymptote
intermediate value theorem	indefinite integral	concavity
mean value theorem for derivatives	inflection points	extreme points
1st order critical points	extreme value theorem	function
fundamental theorem of calculus	2nd order critical points	

Survey Of Geometries

Euclidean	4-point	SMSG	finite	unique parallel line
neutral	Hilbert	affine	4-line	no parallel lines
incidence	3-point	Fano's	elliptic	multiple parallel lines
Young's	hyperbolic	Birkhoff	projective	axiomatic system

Sets And Functions

set	equal	empty	proper	ordered pairs
subset	union	intersection	complement	Cartesian product
relation	function	domain	symmetric	reflexive
range	element	transitive	equivalent	1-1 correspondence

Number Theory

factor	factor tree	composite	intersection of sets
remainder	prime	divisor	least common multiple
factorization	multiple	product	division by primes
integer	even	relatively prime	Euclidean algorithm
odd	divisible	prime factorization	greatest common divisor

Appendix C: Excerpts from Essays Corresponding to Sample Concept Maps

#1 Calculus I: Functions: *The Function That Could*

This map is the story of a function. Functions are found on the planet Calculus. Functions have to behave in certain ways or they will be exiled off of the planet.... The job of a function is to eat a x (function food) from a grocery store A ($x \in A$) and to spit it out to form only one $f(x)$(function spit) into a sink B ($f(x) \in B$). The grocery store A is called the domain, because it contains all possible x's to be eaten. The sink B is called the range, because it contains one value $f(x)$ for each x eaten.... Every function is unique, just like people. On the planet Calculus there are different races of functions. These races are defined by their own equation. Some races are as follows: piecewise, trigonometric, quadratic, constant, linear, power, exponential, and logarithmic.... There comes a time in a function's life when it must find a mate. Functions are neither male nor female, and aren't too picky. When two functions marry, it is called composition of the functions ($f(g(x))$ or $f \circ g$). This is defined by putting the value of $g(x)$ into the $f(x)$ to make a new functions. (This is how functions evolve on the planet Calculus)....

The world of Calculus is a complex thing. If you're thinking of traveling there, take Calculus first so you will know your way around.

#2 Calculus I: Differential and Integral Calculus

The central theme that I used to relate the different terms was the *Fundamental Theorem of Calculus*, which combines both the *derivative* and the *Indefinite integral*.

The *derivative* is the *instantaneous rate of change* of a function or graphically it is the *slope of the tangent line*. The *slope of the tangent line* is the *limit* of the slopes of secant lines....The *derivative* of the *position function* is the *velocity function* and the *derivative* of the *velocity function* is the *acceleration function*. The *derivative* does not

exist at points where the vertical *asymptotes* exist. (I didn't show this relationship on the concept map to avoid extra lines.)....

The *indefinite integral* is an *antiderivative* of a *function*....This is part of the *fundamental theorem of calculus*....The *indefinite integral* is a *function,* where the *definite integral* is a number which represents the *area under the curve* of a positive *function*. The *definite integral* is the limit of *Riemann sums*....

#3 Survey of Geometries: *County of Axiomatic Systems*

I chose an approach to my concept map that parallels our trip this quarter to the many roads we have explored in the city of geometry. [The title of our text was *Roads to Geometry*.] Everything we have worked with this quarter has been part of some sort of axiomatic system. That is why everything is contained in the large box. In this county (like most counties) exists a city; the city of geometry to be exact. Of course there are other cities in this county such as calculus, trigonometry, etc. But I am only concerned with the city of geometry.....This is just a map of all the major routes to various destinations. (For short cut directions, consult a local geometry student. Be careful however, I once consulted a student for a shorter way and I ended up in the wrong section of town.)

#4 Structure of Mathematics I: Number Theory

I found that this map was a lot harder to construct than our previous map. I used the greatest common divisor and the least common multiple as my starting points. Prime was also a very important term that related to many of the other terms. I used

the terms definitions and relationships to decide the similarities. I also included the terms purposes to make my connections between words....

A factor tree is a way of finding the prime factorization so I connected it [to prime factorization]. Moving down from factor tree is the term factor which are the components of a factor tree. Product and factorization were also included because factors are a product of factorization....Relatively prime joins with prime because relatively prime numbers have a GCD of one which is prime.

The map really helped me make these concepts more concrete in my mind. Like the previous map this map was very helpful for making similarities and differences between all of the terms. I wish we could make a map before every test that way all the terms would be truly understood.

Appendix D: Holistic Scoring Criteria

Concept Map

Organization: description of clusters and connections used

6 — Excellent: shows complete, in-depth understanding of links among various terms; creates clear and insightful clusters of related terms; utilizes exemplary linking words; may add terms

5 — Fluent: shows a thorough understanding of links among various terms; creates illustrative clusters of related terms; utilizes effective linking words; uses all terms

4 — Good: shows a general understanding of links among various terms; creates adequate clusters of related terms; utilizes applicable linking words; may omit a few terms

3 — Fair: shows a partial understanding of links among various terms; creates understandable clusters of related terms; utilizes adequate linking words; may omit some terms

2 — Weak: shows a minimal understanding of links among various terms; creates deficient clusters of related terms; utilizes unsuitable linking words; omits several key terms

1 — Inadequate: shows little understanding on links among various terms; creates effective clusters of related terms; utilizes inapplicable/no linking words; omits numerous key terms

0 — Unacceptable: no attempt made or unintelligible

Accuracy: evidence of inaccuracies/misconceptions

4 — Excellent: no errors

3 — Fluent: few minor errors, no conceptual errors

2 — Good: some errors

1 — Weak: numerous errors

0 — Inadequate: numerous major conceptual errors

Interpretive Essay

Communication: clarification of understandings and expression of mathematical ideas

6 — Excellent: demonstrates interpretations and understandings in a clear, systematic, and organized manner; represents mathematical ideas accurately in an exemplary manner

5 — Fluent: demonstrates interpretations and understandings in a clear and organized manner; represents mathematical ideas in an effective manner; may contain minor misconceptions

4 — Good: demonstrates interpretations and understandings in an organized manner; represents mathematical ideas in a proficient manner; may contain some misconceptions

3 — Fair: demonstrates interpretations and understandings in an understandable manner; represents mathematical ideas in an acceptable manner; may contain several misconceptions
2 — Weak: demonstrates interpretations and understandings in a manner that is disorganized or difficult to understand; represents mathematical ideas in an inappropriate manner; may contain numerous misconceptions
1 — Inadequate: demonstrates interpretations and understandings in a manner that is impossible to understand; represents mathematical ideas in an inaccurate manner; numerous misconceptions
0 — Unacceptable: no attempt made or unintelligible

Organization
3 — Excellent: method of presentation clear and appropriate transitions
2 — Adequate: relationships and transitions sometimes unclear
1 — Poor: lacks coherence; disjointed statements
0 — Unacceptable: no attempt made or unintelligible

Mechanics
1 — Acceptable: few violations in grammar, punctuation, capitalization
0 — Unacceptable: errors interfere with understanding; unintelligible

AREA 1997

SE060194

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement (OERI)
Educational Resources Information Center (ERIC)

ERIC®

REPRODUCTION RELEASE

(Specific Document)

I. DOCUMENT IDENTIFICATION:

Title: ASSESSING MATHEMATICAL KNOWLEDGE WITH CONCEPT MAPS AND INTERPRETIVE ESSAYS	
Author(s): LINDA A. BOLTE	
Corporate Source: EASTERN WASHINGTON UNIVERSITY	Publication Date: MARCH 24, 1997

II. REPRODUCTION RELEASE:

In order to disseminate as widely as possible timely and significant materials of interest to the educational community, documents announced in the monthly abstract journal of the ERIC system, *Resources in Education* (RIE), are usually made available to users in microfiche, reproduced paper copy, and electronic/optical media, and sold through the ERIC Document Reproduction Service (EDRS) or other ERIC vendors. Credit is given to the source of each document, and, if reproduction release is granted, one of the following notices is affixed to the document.

If permission is granted to reproduce the identified document, please CHECK ONE of the following options and sign the release below.

[X] Sample sticker to be affixed to document	Sample sticker to be affixed to document [ ]
Check here Permitting microfiche (4"x 6" film), paper copy, electronic, and optical media reproduction	**or here** Permitting reproduction in other than paper copy.
"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY ____ Sample ____ TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."	"PERMISSION TO REPRODUCE THIS MATERIAL IN OTHER THAN PAPER COPY HAS BEEN GRANTED BY ____ Sample ____ TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."
Level 1	**Level 2**

Sign Here, Please

Documents will be processed as indicated provided reproduction quality permits. If permission to reproduce is granted, but neither box is checked, documents will be processed at Level 1.

"I hereby grant to the Educational Resources Information Center (ERIC) nonexclusive permission to reproduce this document as indicated above. Reproduction from the ERIC microfiche or electronic/optical media by persons other than ERIC employees and its system contractors requires permission from the copyright holder. Exception is made for non-profit reproduction by libraries and other service agencies to satisfy information needs of educators in response to discrete inquiries."

Signature: Linda A. Bolte	Position: ASSISTANT PROFESSOR
Printed Name: LINDA A. BOLTE	Organization: EASTERN WASHINGTON UNIVERSITY
Address: MATHEMATICS DEPT MS #32 EASTERN WASHINGTON UNIVERSITY CHENEY, WA 99004	Telephone Number: (509) 359-6071
	Date: March 31, 1997

OVER

CUA

THE CATHOLIC UNIVERSITY OF AMERICA
Department of Education, O'Boyle Hall
Washington, DC 20064
202 319-5120

February 21, 1997

Dear AERA Presenter,

Congratulations on being a presenter at AERA[1]. The ERIC Clearinghouse on Assessment and Evaluation invites you to contribute to the ERIC database by providing us with a printed copy of your presentation.

Abstracts of papers accepted by ERIC appear in *Resources in Education (RIE)* and are announced to over 5,000 organizations. The inclusion of your work makes it readily available to other researchers, provides a permanent archive, and enhances the quality of *RIE*. Abstracts of your contribution will be accessible through the printed and electronic versions of *RIE*. The paper will be available through the microfiche collections that are housed at libraries around the world and through the ERIC Document Reproduction Service.

We are gathering all the papers from the AERA Conference. We will route your paper to the appropriate clearinghouse. You will be notified if your paper meets ERIC's criteria for inclusion in *RIE*: contribution to education, timeliness, relevance, methodology, effectiveness of presentation, and reproduction quality. You can track our processing of your paper at http://ericae2.educ.cua.edu.

Please sign the Reproduction Release Form on the back of this letter and include it with **two** copies of your paper. The Release Form gives ERIC permission to make and distribute copies of your paper. It does not preclude you from publishing your work. You can drop off the copies of your paper and Reproduction Release Form at the **ERIC booth (523)** or mail to our attention at the address below. Please feel free to copy the form for future or additional submissions.

Mail to: AERA 1997/ERIC Acquisitions
The Catholic University of America
O'Boyle Hall, Room 210
Washington, DC 20064

This year ERIC/AE is making a **Searchable Conference Program** available on the AERA web page (http://aera.net). Check it out!

Sincerely,

Lawrence M. Rudner, Ph.D.
Director, ERIC/AE

[1]If you are an AERA chair or discussant, please save this form for future use.